Kuwait University
Dept.of Maths.&Comp. Sci.

Math. 101
First Exam
Make-up.

Date : October 31st, 2000
Duration : 75 minutes.

Calculators Mobile Telephones and Pagers are not allowed.
Answer all the following questions. Show your work.

1. Use the definition of the limit to show that

$$\lim_{x\to\frac{2}{3}} (4-5x) = \frac{2}{3}.$$ (3 pts.)

2. Find the following limits, if they exist:

(a) $\lim_{x\to 0} \dfrac{\sqrt{x+9} - \sqrt[3]{x+27}}{x}$ (3 pts.)

(b) $\lim_{x\to 0} \dfrac{1-\cos 2x}{\sin^2 x}$ (3 pts.)

(c) $\lim_{x\to 1} \dfrac{\sqrt{|x-1|}}{x-1}$ (3 pts.)

(d) $\lim_{x\to-\infty} \dfrac{\sqrt{x^6+1}}{|x^3+x|}$ (3 pts.)

3. Find the vertical and horizontal asymptotes, if any, of the graph of the function

$$f(x) = \begin{cases} \dfrac{\sin x}{x^2}, & \text{if } x > 1 \\ \dfrac{|x|+1}{x+1}, & \text{if } x \le 1. \end{cases}$$ (3 pts.)

4. Classify the discontinuities of f as removable, jump, or infinite, where

$$f(x) = \frac{(x+1)\,|x-2|\,\sqrt{|x-1|}}{x^3 - 2x^2 - x + 2}$$ (4 pts.)

5. Use the intermediate value theorem to prove that any polynomial of odd degree should have at least one real root. (3 pts.)

Good Luck